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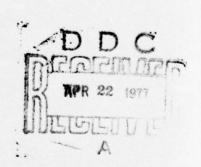
## A WIDEBAND WAVEGUIDE LENS

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#### ABSTRACT

A wideband compound waveguide lens which is the analog of the optical achromatic doublet is described and analyzed for its phase error characteristics. It is shown that the bandwidth of the compound lens varies from 40 to 20 percent for lens diameters varying from 20 to 100 wavelengths. This is an order of magnitude greater than the bandwidth heretofore obtainable with conventional waveguide lenses.

Oliste Sanda Anala Maria

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## A Wideband Waveguide Lens

#### I. Introduction

Waveguide lenses are generally zoned to increase their bandwidths. The diameters of the zones are chosen to satisfy specific requirements; of particular interest are the two cases where the diameters are chosen to either minimize the thickness of the lens [1] or to minimize the phase error over a frequency band [2]. The minimum-phase-error zoned lens has more zones than the minimum-thickness zoned lens but it exhibits a much smaller phase error than the latter one. A different type of waveguide lens is the constantthickness lens wherein the phase correction of off-axis rays is achieved by means of phase shifters inserted in the waveguide elements. A practical wideband phase shifter for this application is the half-wave plate which causes the phase of a circularly polarized wave incident upon it to be shifted in accordance with the plate orientation [3]. The differential phase shift is equal to twice the differential rotation of the plate and therefore is independent of frequency, provided the half-wave plate is broadband. Because the phase shift is constant with frequency the constant-thickness, half-wave-plate lens is narrowband. In this report a waveguide lens exhibiting a much larger bandwidth than the above lenses is described. lens is a compound lens obtained by combining a conventional waveguide lens with a constant-thickness, variable-phase-shift lens and it does not require zoning. The compound waveguide lens is analogous to the optical achromatic doublet. The design equations, the phase error function and the bandwidth of the compound lens are derived in the next Section and a particular case

considered which shows the phase error of the compound lens to be an order of magnitude smaller than that of the minimum-phase-error zoned lens.

## II. Compound Lens Analysis

Consider the waveguide lens of Fig. 1 where each waveguide element consists of a section of length L followed by a section of length T; the latter contains a phase shifter with phase shift  $\phi$  independent of frequency. T is the same for all elements but L is a function of the element displacement r. Letting the length of the center element be  $L_0$  and the differential phase caused by the phase shifter in this element be zero (i.e.,  $\phi$ =0) and considering the case where the radius of the inner spherical surface of the lens is equal to the focal length, the equiphase condition is satisfied when

$$2\pi nL/\lambda + \phi + 2\pi t/\lambda = 2\pi nL_0/\lambda \tag{1}$$

where n =  $[1-(\lambda/\lambda_c)^2]^{1/2}$  and  $\lambda_c$  is the waveguide cutoff wavelength. Referring to Fig. 1 it is seen that t =  $L_o$  - L + s and therefore (1) can be expressed as

$$(1-n) (L-L_0) - \phi \lambda / 2\pi - s = 0$$
 (2)

Since two parameters need to be determined, Eq. (2) may be satisfied at two frequencies or it may be satisfied at a single frequency and a second equation obtained by making (2) stationary with respect to frequency. These two cases are studied separately.

#### 1. Single-Frequency Design

With  $\lambda_0$  the design free-space wavelength and  $n_0$  the corresponding index of refraction (2) becomes

$$(1-n_0) (L-L_0) - \phi \lambda_0 / 2\pi - s = 0$$
 (3)

Making (3) invariant with respect to the free-space wavelength  $\lambda_{o}$  leads to

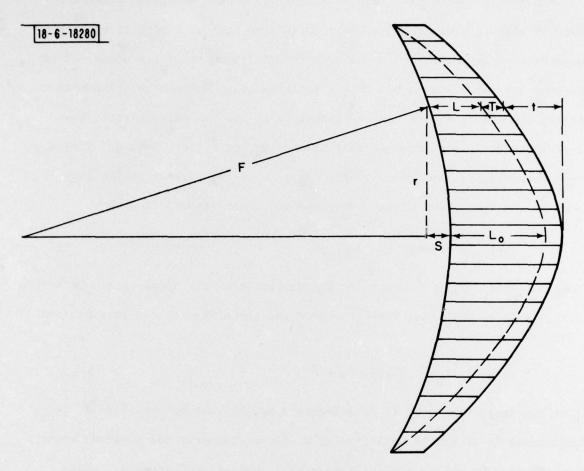


Fig. 1. Compound lens

$$(L-L_o) \frac{dn_o}{d\lambda_o} + \phi/2\pi = 0$$
 (4)

Since

$$n_o = [1 - (\lambda_o/\lambda_c)^2]^{1/2}$$
 (5)

$$\frac{dn_o}{d\lambda_o} = (n_o^2 - 1)/n_o \lambda_o . ag{6}$$

Substituting (6) in (4) and solving, there results

$$L = L_{0} - n_{0} s/(1-n_{0})$$
 (7)

and 
$$\phi = -2\pi s (1+n_o)/\lambda_o + 2m\pi$$
 (8)

where m is an integer. For a lens of diameter D and with inner spherical surface of radius equal to the focal length F, the variable s is related to the radial distance r of a waveguide element by

$$s = F - [F^2 - f^2]^{1/2}$$
 (9)

and

$$s_{MAX} = F - [F^2 - D^2/4]^{1/2}$$
 (10)

The thickness of the lens at the center is obtained from (7) by making L=0 for  $s=s_{MAX}$  yielding

$$L_{o} = n_{o} s_{MAX} / (1 - n_{o}) \tag{11}$$

The outer surface of the lens is convex as illustrated in Fig. 1. At the

design frequency the lens is free of phase error and remains so for small deviations from this frequency. For larger deviations the phase error, i.e., the phase difference between a general ray and the center ray is

$$\phi_{e} = 2\pi n L/\lambda + \phi + 2\pi t/\lambda - 2\pi n L_{o}/\lambda = -\frac{2\pi}{\lambda} (L-L_{o})(1-n) + \frac{2\pi s}{\lambda} + \phi$$

where  $\lambda$  and n are, respectively, the free-space wavelength and the index of refraction at the operating frequency. Substituting L and  $\phi$  from (7) and (8) into (12) the phase error of a center-frequency-designed compound lens becomes:

$$\phi_{e} = \frac{2\pi s}{\lambda_{o}} \left[ \left( \frac{\lambda_{o}}{\lambda} \right) \left( \frac{1 - n_{o}}{1 - n_{o}} \right) - n_{o} - 1 \right]$$
(13)

The normalized phase error  $\phi_e/(2\pi s/\lambda_o)$  is plotted in Fig. 2 as a function of  $f/f_o$ , for a practical value of the design index of refraction  $(n_o = 0.6)$ . Calculations indicate that the phase error decreases monotonically as  $n_o$  increases and that  $\Delta\phi_e/\phi_e \simeq -\Delta n_o/n_o$ . The bandwidth defined as the band over which the phase error does not exceed  $\pi/4$  (corresponding to wavefront deviations of  $\pm \lambda/16$ ) is given in Fig. 3 as a function of  $D/\lambda_o$ . The bandwidth of the minimum-thickness zoned waveguide lens also shown in Fig. 3 is an order of magnitude smaller than that of the compound lens.

#### 2. Two-Frequency Design

For the lens to be free of phase error at two frequencies Eq. (2) must be satisfied at each of these two frequencies, or

$$(1-n_1) (L-L_0) - \phi \lambda_1/2\pi - s = 0$$
 (14)

and

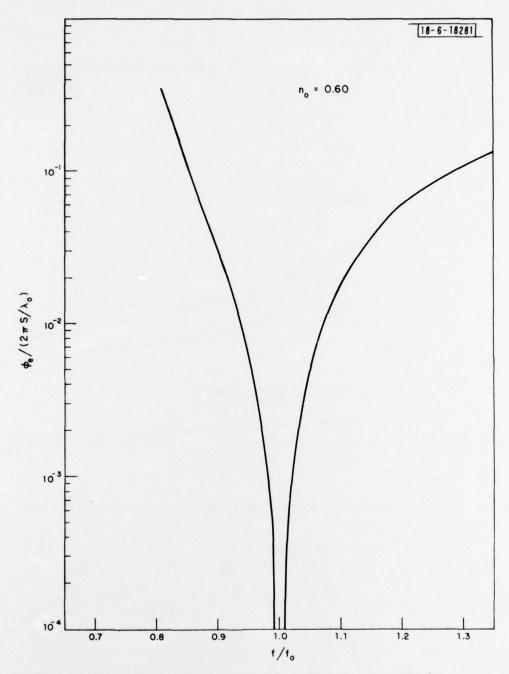


Fig. 2. Normalized phase error of compound waveguide lens.

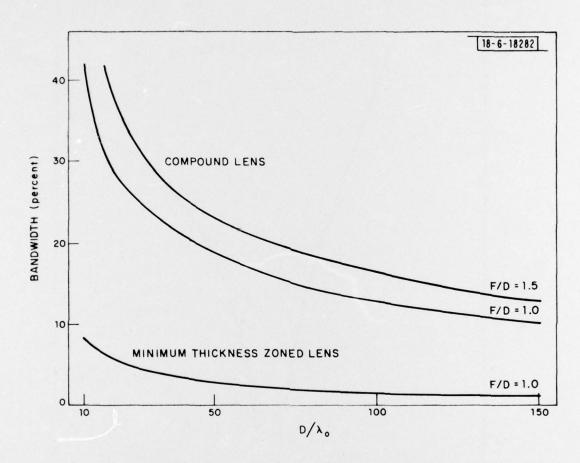


Fig. 3. Bandwidth of compound lens for center-frequency design.

$$(1-n_2) (L-L_0) - \phi \lambda_2/2\pi - s = 0$$
 (15)

where  $\lambda_1$ ,  $\lambda_2$  are the free-space wavelengths and  $n_1$ ,  $n_2$  are the indices of refraction corresponding to the design frequencies. Solving

$$L = L_0 + \frac{s(\lambda_1 - \lambda_2)}{(1 - n_2)^{-\lambda_1} - (1 - n_1)^{-\lambda_2}}$$
(16)

and

$$\phi = \frac{2\pi (n_2 - n_1)s}{(1 - n_2)^{\lambda_1} - (1 - n_1)^{\lambda_2}} + 2m\pi$$
 (17)

At a different free-space wavelength,  $\lambda$ , the phase error obtained by substituting (16) and (17) in (12) is

$$\phi_{e} = \frac{2\pi s}{\lambda} \left[ \frac{\lambda_{2}(n_{1}-n) - \lambda_{1}(n_{2}-n) - \lambda(n_{1}-n_{2})}{(1-n_{2})\lambda_{1} - (1-n_{1})\lambda_{2}} \right]$$
(18)

The phase error for a given waveguide element reaches a maximum at a free-space wavelength obtained by making  $d\phi_e/d\lambda$  = 0 yielding

$$n = (\lambda_2 - \lambda_1) / (n_1 \lambda_2 - n_2 \lambda_1)$$
 (19)

to which corresponds

$$\lambda = \lambda_1 [(n^2 - 1)/(n_1^2 - 1)]^{1/2}$$
 (20)

# III. Comparison of Single- and Double-Frequency Designs

The phase error at the edge of a compound lens with D = 46 inches, F/D = 1 and  $n_0 = 0.62$  is plotted in Fig. 4 as a function of frequency. The top curve applies to a single-frequency design at  $f_0 = 8.15$  GHz and the bottom curve applies to a two-frequency design with  $f_1 = 8.0$  GHz and  $f_2 = 8.3$  GHz. The bottom curve is very nearly identical to the top curve but displaced by an amount equal to the phase error of the single-frequency design at  $f_1$  and  $f_2$ . Thus the optimum design frequencies for the two-frequency case may be determined from the phase error curve of the single-frequency design. The bandwidth of the two-frequency design is plotted in Fig. 5 as a function of  $D/\lambda_0$  for F/D = 1. The bandwidth is seen to vary from about 40 to 20 percent for lenses with diameters varying from 20 to 100 wavelengths. The bandwidth of the two-frequency design is about 40% greater than that of the single-frequency design.

For comparison the phase error of the two types of zoned lenses and of the constant-thickness, variable-phase-shift lens are plotted as a function of  $s/\lambda_0$  in Fig. 6, together with that of the single-frequency-design compound lens. The design frequency and index of refraction for all lenses are identical, namely 8.15 GHz and 0.62, respectively, and the phase error is calculated for a frequency of 8.0 GHz. The phase error of the compound lens is observed to be an order of magnitude smaller than that of the minimum-phase-error zoned lens.

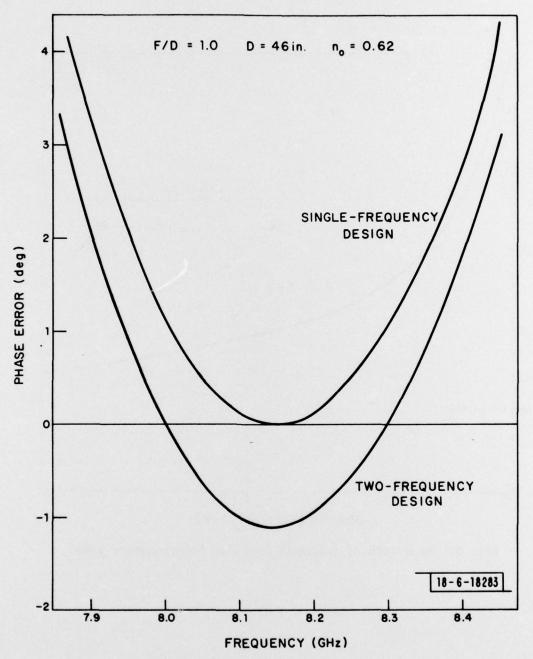


Fig. 4. Phase error of single- and two-frequency designs.

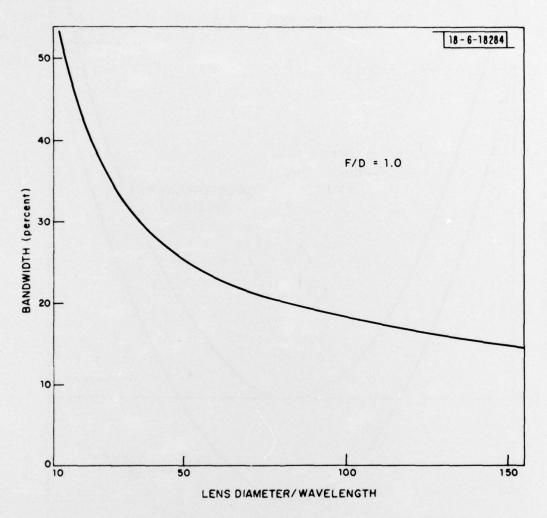


Fig. 5. Bandwidth of compound lens for two-frequency lens.

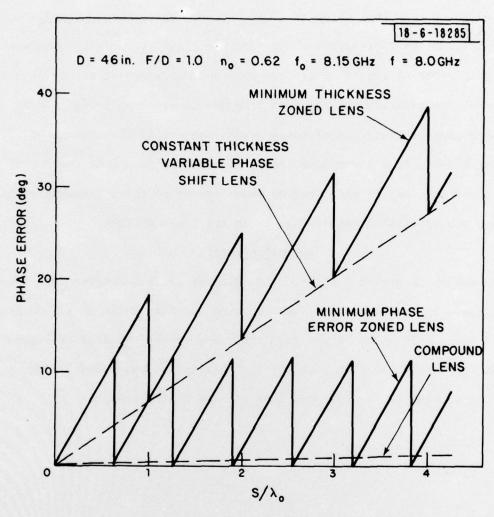


Fig. 6. Phase error of waveguide lenses.

#### IV. Conclusions

A compound lens consisting of a conventional waveguide lens and a constant-thickness, variable-phase-shift lens was shown to exhibit a bandwidth which is an order of magnitude greater than the bandwidth of either of its components. In addition the compound lens may be expected to exhibit an efficiency greater than that of zoned lenses because of the absence of shadowing effects in the compound lens. Finally, it should be noted that the compound lens may be made free of phase errors at three frequencies by releasing the constraint imposed herein on the inner surface.

#### Acknowledgements

The author is grateful to Dr. L. J. Ricardi for his interest and valuable suggestions. The author also wishes to point out that while this Technical Note was being written he became cognizant of a recent proposal by Hughes Aircraft Corp. in which, interestingly, a description was made of a waveguide lens similar to the compound lens treated in this report.

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